# A list of 43 Double Compact Stars

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**Abstract**

A star at the end stage of its evolution will lead to the formation of a compact object such as a White Dwarf (WD), a Neutron Star (NS) or a Black Hole (BH). Since a significant fraction of the stars that we observe in our Galaxy are part of close binary systems it is plausible to expect also an important fraction of binary systems in which the two components had already attained the compact stage. In this work we have elaborated a list of 43 Double Compact Stars (DCSs) including WD-WD, WD-NS, NS-NS and BH-BH systems and explored possible relations between masses, orbital periods ($P_{orb}$) and orbital semi-major axis ($a$). As a result we found out the empirical relation $P_{orb} \approx \frac{a^{1.5}}{10}$ which is in fact an expression of Kepler's Third Law.

***Keywords:*** (stars:) binaries (including multiple): close, stars: neutron, (stars:) pulsars: general, (stars:) white dwarfs, stars: black holes

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# 1 Introduction

The end product of stellar evolution of a particular star depends on its respective initial mass. A star with an initial mass above $0.8M_{\odot}$ will end up as a compact object (e.g. Sobrinho, 2003). Depending on the initial mass of the star, this compact object could be a White Dwarf (WD), a Neutron Star (NS) or a Black Hole (BH). A star with an initial mass of $0.8 - 8M_{\odot}$, will end up as a WD with a mass that could be as low as $0.17M_{\odot}$, such as SDSS J0917+46, one of the lowest mass WD known to date (cf. Kilic et al., 2007), and as high as $1.4M_{\odot}$ (the Chandrasekhar limit which corresponds to the maximum mass allowed for a stable WD). If the initial mass of the star is around $8 - 25M_{\odot}$, then the final product will probably be a NS with a mass typically within the range $1.4 - 3M_{\odot}$ (Heger et al., 2003). With $2.35M_{\odot}$, PSR J0952-0607 is one of the most massive NSs known to date (Romani et al., 2007). A number of NSs present in binary systems have measured masses $< 1.4M_{\odot}$ (i.e. bellow the Chandrasekhar limit). This is, for example, the case of J0453+1559 a binary with a $1.17M_{\odot}$ NS (Martinez et al., 2015)[2]. If the initial mass of the star is $> 25M_{\odot}$, then the final product will probably be a BH with a mass greater than $3M_{\odot}$ (Heger et al., 2003).

It appears that approximately half of all stars in our galaxy are in binary systems (e.g. Kutner, 2003). In many of the observed cases, only one of the stars is in fact visible. Depending on how the companion star manifests its presence, we can classify binaries in different categories such as: optical double, visual binary, composite spectrum binary, eclipsing binary, astrometric binary, spectroscopic binary (see e.g. Kutner, 2003, for more details).

If the distance between the two components on a binary system is large ($> 200R_{\odot}$) then we say that we have a wide binary (e.g. Green & Jones, 2004). In this case, there will be little interaction between the stars apart from their mutual gravitational influence on one another. On the other hand, if the distance is relatively short in such a way that one star can gravitationally distort the other, then we have a **close binary system** (e.g. Kutner, 2003). In certain situations, it is possible for material from one star to be pulled onto the other star. This process of mass transfer has direct observational consequences and, over time, the evolution of both stars can be significantly modified if compared to the evolution process of similar individual stars (e.g. Green & Jones, 2004).

Given the number of binary stars in our galaxy it is expected that a relevant number of binary systems in which both stars have already reached the endpoint of their evolution (WD, NS, BH) and that some of these systems can be categorized as close binaries. In this work, we compiled a sample of those close compact double binary systems and explored some of the parameters characterizing them.

[2] Notice that the Chandrasekhar limit refers only to the maximum mass allowed for a WD and not for the minimum mass allowed for a NS. In fact, depending on the stellar evolution pathway for a particular star (or binary star) we may get at the endpoint a NS with less than $1.4M_{\odot}$.

The paper is organized as follows: in Section 2 we review some topics on binary systems and in Section 3 we present a sample of compact double binaries. In Section 4 we explore to some extent the data presented in our sample. Finally, in Section 5 we present some conclusions.

# 2 Binary Systems

Given a binary system with two compact objects with masses $M_1$ and $M_2$ we will consider, without any loss of generality, that $M_1 \geq M_2$ and we will refer to the object with mass $M_1$ as the primary component and to the object with mass $M_2$ as the secondary component. The total mass of the system is given by:

$$M = M_1 + M_2 \tag{1}$$

and we will define the mass ratio by:

$$q = \frac{M_2}{M_1} \tag{2}$$

Recalling that we are assuming $M_1 \geq M_2$, we will always get $q \in ]0, 1]$ [3].

In a binary system, the two stars describe elliptical orbits around their common center of mass. From the definition of center of mass, we have (e.g. Kutner, 2003):

$$M_1 a_1 = M_2 a_2 \tag{3}$$

where ($M_1$,$M_2$) represents the masses of the stars and ($a_1$,$a_2$) are the semi-major axis of their elliptical orbits. Since the center of mass must always be along the line joining the two stars, the stars must always be on opposite sides of the center of mass. This means that the two stars share the same orbital period $P_{orb}$ and that the distance between them is always given by (e.g. Kutner, 2003):

$$a = a_1 + a_2 \tag{4}$$

In the case of a BH-BH system the Innermost Stable Circular Orbit (ISCO) is defined as the last complete orbit before the transition to the merger stage in which the two BHs merge into a single BH with the emission of an appreciable amount of energy in the form of GWs (Buonanno et al., 2008). The ISCO is located at (Kagohashi et al., 2024):

$$r_{ISCO} = \frac{6GM}{c^2} \tag{5}$$

[3] Some authors consider instead $q = \frac{M_1}{M_2}$ and in that case one would always get $q \geq 1$.

where $M$ is the mass of the BH. Assuming that the distance $a$ is sufficiently enough in order to avoid significant relativistic effects, the motion of the stars can be described by means of Kepler's Third Law (e.g. Kutner, 2003):

$$M_1 + M_2 = \frac{4\pi^2}{G} \frac{a^3}{P_{orb}^2} \tag{6}$$

Taking into account that in the center of mass reference frame the relation given by equation (3) is valid, we may write Kepler's Third Law as:

$$\frac{M_2^3}{(M_1 + M_2)^2} = \frac{4\pi^2}{G} \frac{a_1^3}{P_{orb}^2} \tag{7}$$

If one considers the velocity $v_1$ of the star of mass $M_1$ around the center of mass, we may write equation (7) in the form (e.g. Sobrinho, 2003)

$$\frac{M_2^3}{(M_1 + M_2)^2} = \frac{v_1^3 P_{orb}}{2\pi G} \tag{8}$$

In practice, it is easier to measure the radial component ($v_{1r}$) of the velocity ($v_1$). If $i$ is the angle between the line of sight and the orbital plane of the binary system, we may write (e.g. Sobrinho, 2003):

$$\frac{(M_2 \sin(i))^3}{(M_1 + M_2)^2} = \frac{v_{1r}^3 P_{orb}}{2\pi G} \tag{9}$$

The left hand side of equation (9) defines the so-called *mass function* $f(M)$. Setting $i = \frac{\pi}{2}$ and $M_1 = 0$ such that $f(M) = M_2$, then the right hand side of equation (9) gives us the lower limit for $M_2$ (e.g. Sobrinho, 2003).

It may be useful for our own purposes to write Kepler's Third Law with the masses in units of $M_\odot$, the time in days, and the distances in units of $R_\odot$. From equation (6), we get:

$$M_1 + M_2 = \left(\frac{4\pi^2}{G}\right) \left(\frac{1R_\odot^3}{(1\mathrm{day})^2 \cdot 1M_\odot}\right) \left(\frac{a^3}{P_{\mathrm{orb}}^2}\right) \tag{10}$$

Replacing the values of all the constants in equation (10), we get:

$$M_1 + M_2 \approx \frac{1}{75} \left(\frac{a^3}{P_{\mathrm{orb}}^2}\right) \tag{11}$$

In the rest of this work, unless stated otherwise, we will always assume that masses are expressed in solar masses, distances in solar radii and time intervals in days.

# 3 A sample of double compact binaries

In this section, we compile some observational and derived parameters on 43 Double Compact Systems, most of them close binary systems.

In Tables 1 and 3 we show the masses (including individual masses ($M_1$ and $M_2$), total mass $M$ and mass ratio $q$ - cf. equation 2), orbital period $P_{orb}$, and the orbit semi-major axis $a$ for, respectively, 14 WD-WD and 11 WD-NS binaries. Notice that in Table 1, we have determined for some of the cases the value of the semi-major axis $a$ by using equation (11) since this value was not shown in the consulted references.

In Table 2, we present the values for 8 NS-NS binaries. Besides the mass related parameters (individual masses, total mass and mass ratio), orbital period, and the orbit semi-major axis, we also present the values for the mass function $f(m)$ and for the inclination angle ($i$).

Finally, in Table 4, we show the results for 10 BH-BH binary systems. So far, the stellar mass BH-BH pairs identified in nature are those related to Gravitational Wave (GW) emission after a coalescence and merging process. In fact, when this GWs where detected, the two BHs are already at the end stage of the merging process meaning that the binary system no longer exists. For our purposes, we have considered the 10 binary BH mergers with the lowest total mass from Freitas & Sobrinho (2021). For each case we have considered a semi-major axis given by:

$$a = 1000(a_{ISCO1} + a_{ISCO2}) \tag{12}$$

where $a_{ISCO1}$ and $a_{ISCO2}$ (cf. equation 5) represent the Innermost Stable Circular Orbit for each BH of a certain binary system (e.g. Freitas & Sobrinho, 2021). The value of the orbital period was determined inserting into equation (6) the value of $a$ obtained from equation (12) with the help of equation (5).

Table 1: A list of WD-WD binary systems. For each case, it is shown the mass of the primary WD ($M_1$), the mass of the secondary WD ($M_2$), the total mass ($M$), the mass-ratio ($q$), the orbital period ($P_{orb}$) and the semi-major axis of the orbit ($a$). The values of the masses with the label $*$ correspond to the lower limit. $**$ indicates that the semi-major axis $a$ was determined using Kepler's Third Law (see equation 6). References: [1] (Maxted et al., 2002); [2] (Han Z., 1998); [3] (Van der Sluys et al. , 2006); [4] (Nelemans et al., 2005)

| Name | $M_1(M_\odot)$ | $M_2(M_\odot)$ | $M(M_\odot)$ | $q$ | $P_{orb}$ (days) | $a(R_\odot)$ | Ref |
|---|---|---|---|---|---|---|---|
| WD 1101+364 | 0.33 | 0.29 | 0.62 | 0.88 | 0.145 | 0.99 | [1],[3] |
| WD 1713+332 | 0.35 | 0.18* | 0.53 | 0.51 | 1.120 | 3.67** | [2] |
| WD 0957-666 | 0.37 | 0.32 | 0.69 | 0.86 | 0.061 | 0.58 | [1],[3] |
| WD 1241-010 | 0.37* | 0.31 | 0.68 | 0.83 | 3.350 | 8.30** | [2] |
| WD 1317+453 | 0.42* | 0.33 | 0.75 | 0.78 | 4.800 | 10.88** | [2] |
| WD 1349+144 | 0.44 | 0.44 | 0.88 | 1.00 | 2.120 | 6.66** | [2] |
| WD 0136+768 | 0.47 | 0.37 | 0.84 | 0.79 | 1.407 | 4.98 | [1],[3] |
| WD 1202+608 | 0.49 | 0.25* | 0.74 | 0.51 | 1.490 | 4.96** | [2] |
| WD 1155+166 | 0.52 | 0.43 | 0.95 | 0.83 | 30.090 | 40.02** | [2] |
| WD 1204+450 | 0.52 | 0.46 | 0.98 | 0.88 | 1.603 | 5.72 | [1],[3] |
| WD 0135-052 | 0.52 | 0.47 | 0.99 | 0.90 | 1.556 | 5.63 | [1],[3] |
| WD 1704+481.2 | 0.54 | 0.39 | 0.93 | 0.72 | 0.145 | 1.13 | [1],[3] |
| HE 2209-1444 | 0.58 | 0.58 | 1.16 | 1.00 | 0.280 | 1.90** | [4] |
| HE 1414-0848 | 0.71 | 0.55 | 1.26 | 0.77 | 0.520 | 2.94** | [4] |

Table 2: A list of NS-NS binary systems. For each case, the mass function ($f(m)$), the mass of the primary neutron star ($M_1$), the mass of the secondary neutron star ($M_2$), the total mass ($M$), the mass-ratio ($q$), the orbital period ($P_{orb}$), the semi-major axis of the orbit ($a$), and the inclination angle ($i$) are shown. References: [7] (Wong et al., 2010); [8] (Shao et al., 2014)

| **Name** | $f(m)$ | $M_1(M_\odot)$ | $M_2(M_\odot)$ | $M(M_\odot)$ | q | $P_{orb}$(days) | $a(R_\odot)$ | $i$ ($^o$) | **Ref** |
|---|---|---|---|---|---|---|---|---|---|
| J1829+2456 | 0.29 | 1.28 | 1.25 | 2.53 | 0.98 | 1.176 | 6.36 | 79.48 | [7], [8] |
| J1756-2251 | 0.22 | 1.31 | 1.26 | 2.57 | 0.96 | 0.320 | 2.70 | 64.21 | [7], [8] |
| J0737-3039A | 0.29 | 1.34 | 1.25 | 2.58 | 0.93 | 0.102 | 1.26 | 86.15 | [7], [8] |
| B1534+12 | 0.31 | 1.35 | 1.33 | 2.75 | 0.99 | 0.421 | 3.28 | 78.25 | [7], [8] |
| J1906+0746 | 0.11 | 1.37 | 1.25 | 2.61 | 0.91 | 0.116 | 1.75 | 46.75 | [7], [8] |
| B1913+16 | 0.13 | 1.44 | 1.39 | 2.83 | 0.96 | 0.323 | 2.80 | 46.91 | [7], [8] |
| J1811-1736 | 0.13 | 1.62 | 1.11 | 2.60 | 0.69 | 18.779 | 40.70 | 63.06 | [7], [8] |
| J1518+4904 | 0.12 | 1.69 | 0.94 | 2.63 | 0.56 | 8.634 | 24.70 | 89.90 | [7], [8] |

Table 3: A list of WD-NS binary Systems. For each case, the mass of the neutron star ($M_1$), the mass of the white dwarf ($M_2$), the total mass ($M$), the mass-ratio ($q$), the orbital period ($P_{orb}$), and the semi-major axis of the orbit ($a$) are shown. The values of $a$ were determined using Kepler's Third Law ( equation 6). † indicates values calculated using the mass-ratio $q$ ($M_2/M_1$). References: [5] (Ding et al., 2023); [6] (Yu et al., 2024)

| Name | $M_1(M_\odot)$ | $M_2(M_\odot)$ | $M(M_\odot)$ | $q$ | $P_{orb}$ (days) | $a(R_\odot)$ | Ref |
|---|---|---|---|---|---|---|---|
| J1713+0747 | 1.33 | 0.29 | 1.62 | 0.22 | 67.830 | 16.01 | [5] |
| J2140-2310A | 1.35 | 0.11 | 1.46 | 0.08 | 0.170 | 1.47 | [6] |
| J1701-3006B | 1.35 | 0.14 | 1.49 | 0.10 | 0.140 | 1.30 | [6] |
| J1748-2446N | 1.35 | 0.56 | 1.91 | 0.41 | 0.390 | 2.79 | [6] |
| J1952+2630 | 1.35 | 1.13 | 2.48 | 0.84 | 0.390 | 3.04 | [6] |
| J0348+0432 | 1.40 | 0.10 | 1.50 | 0.07 | 0.100 | 1.04 | [6] |
| J1757-5322 | 1.40 | 0.60 | 2.00 | 0.43 | 0.450 | 3.11 | [6] |
| J1141-6545 | 1.40 | 1.00 | 2.40 | 0.71 | 0.200 | 1.93 | [6] |
| J0437-4715 | 1.45 | 0.22 | 1.67 | 0.16 | 5.740 | 82.22 | [5] |
| J1738+0333 | 1.46 | 0.18† | 1.64 | 0.12 | 0.350 | 3.77 | [5] |
| J1012+5307 | 1.82† | 0.17 | 1.99 | 0.10 | 10.441 | 2.46 | [5] |

Table 4: A list of BH-BH binary systems. For each case, the mass of the primary black hole ($M_1$), the mass of the secondary black hole ($M_2$), the total mass ($M$), the mass-ratio ($q$), the orbital period ($P_{orb}$) and the semi-major axis of the orbit ($a$) are shown ( see text for more details ). References: [9] (Freitas & Sobrinho, 2021)

| **Name** | $M_1(M_\odot)$ | $M_2(M_\odot)$ | $M(M_\odot)$ | q | $P_{orb}(\times 10^{-2}$days) | $a(R_\odot)$ | **Ref** |
|---|---|---|---|---|---|---|---|
| GW190425 | 2.00 | 1.40 | 3.40 | 0.70 | 0.057 | 0.043 | [9] |
| GW190924_021846 | 8.90 | 5.00 | 13.90 | 0.56 | 0.231 | 0.18 | [9] |
| GW170608 | 11.00 | 7.60 | 18.60 | 0.69 | 0.310 | 0.24 | [9] |
| GW190707_093326 | 11.60 | 8.40 | 20.10 | 0.72 | 0.333 | 0.25 | [9] |
| GW190930_133541 | 12.30 | 7.80 | 20.10 | 0.63 | 0.335 | 0.26 | [9] |
| GW190728_064510 | 12.30 | 8.10 | 20.30 | 0.66 | 0.340 | 0.26 | [9] |
| GW190720_000836 | 13.40 | 7.80 | 21.50 | 0.58 | 0.353 | 0.27 | [9] |
| GW151226 | 13.70 | 7.70 | 21.80 | 0.56 | 0.356 | 0.27 | [9] |
| GW190814 | 23.20 | 2.59 | 25.80 | 0.11 | 0.429 | 0.33 | [9] |
| GW190708_232457 | 17.60 | 13.20 | 30.90 | 0.75 | 0.513 | 0.39 | [9] |

# 4 Exploring the data

In Figure 1, we show all the 43 compact double star systems from Tables 1, 2, 3 and 4 on the $(\log_{10}(M_1), \log_{10}(M_2))$ plane. Figure 2 represents a zoom in on the lower masses region of Figure 1, i.e., on the region mostly populated by systems involving WDs and NSs. It is clear from both Figures that the majority of the cases are located quite close to the identity line $M_1 = M_2$. Exceptions are related to NS-WD systems where the mass of the WD is generally smaller than that of the companion NS. However, it is interesting to notice that for a few cases the two masses are quite similar.

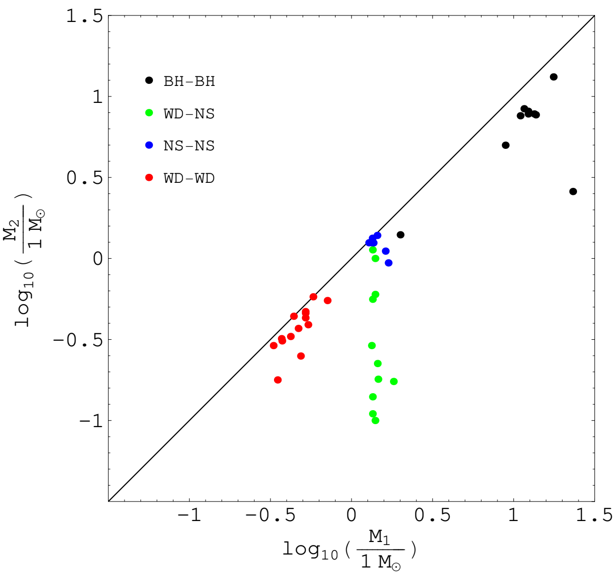


Figure 1: log-log relation between the mass of the primary star and the secondary star of each binary system.

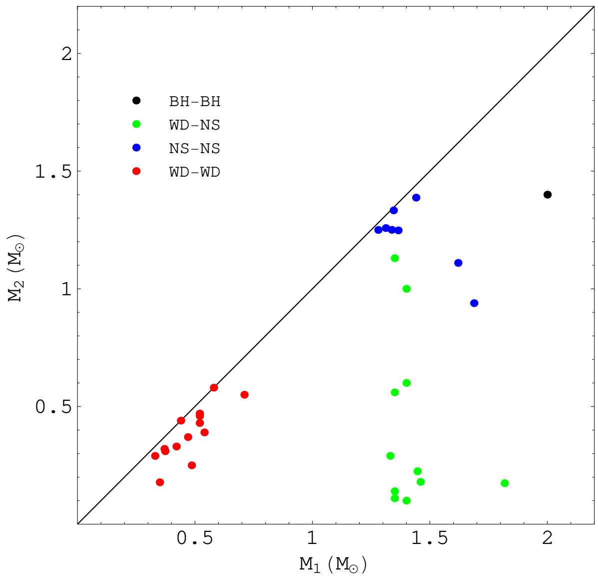


Figure 2: Relation between the mass of the primary star and the secondary star of each binary system (zoom in on the lower mass region of Figure 1).

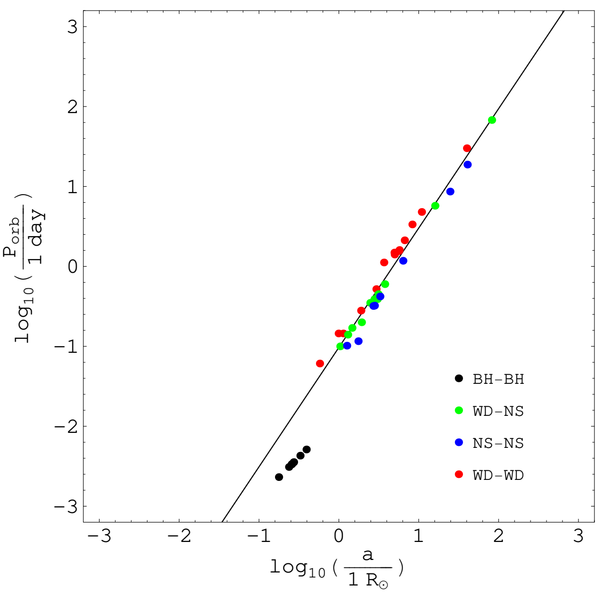


Figure 3: log-log relation between $P_{\rm orb}$ and $a$ for all the 43 double compact binaries from Tables 1, 2, 3 and 4. We performed a linear regression using the Least Squares Method (see text for more details).

In Figure 3, we show the log-log relation between $P_{\rm orb}$ and $a$ for all the 43 double compact binaries from Tables 1, 2, 3 and 4.

Taking into account only the WD-WD, WD-NS and NS-NS pairs we performed a linear regression (using the Least Square Method) from which we obtained the relation:

$$\log P_{orb} \approx 1.5 \log(a^{-1.01}) \tag{13}$$

Solving for $P_{orb}$, we get:

$$P_{orb} \approx \frac{a^{1.5}}{10} \tag{14}$$

which is in agreement with Kepler's Third Law (see equation 11). It is interesting to note that although the systems presented have different natures (NS-NS, WD-WD, NS-WD) and also different total masses, they are arranged along a narrow band in the plane (log $a$, log $P_{orb}$) - see Figure 3. Although, represented in Figure 3, we left the BH-BH systems out of the linear regression, since their arrangement does not seem to respect the same linear relationship as that of the other systems. This discrepancy is probably related to the mass values of these BHs (e.g. $M_1 \in [1.40 M_\odot, 23.20 M_\odot]$).

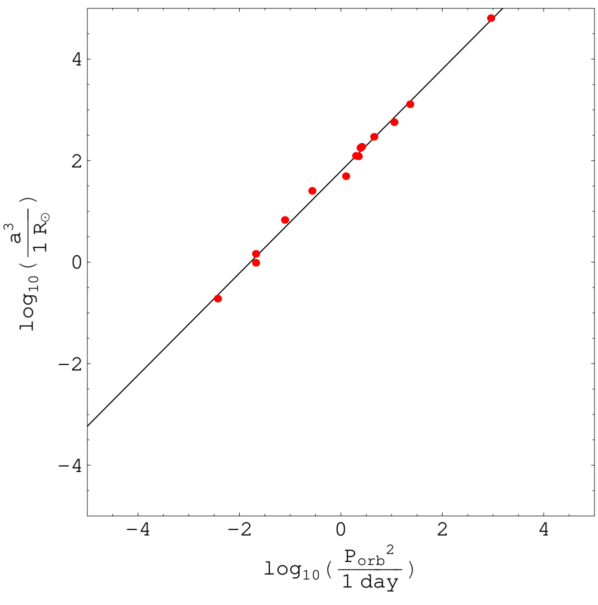


Figure 4: log-log relation between $a^3$ as a function of $P^2_{\rm orb}$ for all the 14 WD-WD pairs presented in Table 1.

In Figure 4, we show $a^3$ as a function of $P^2_{\rm orb}$ for all the 14 WD-WD pairs presented in Table 1. Performing a linear regression, we get the relation:

$$\log a^3 = \log P_{\rm orb}^2 + 1.79 \qquad (15)$$

from which we get:

$$a^3 \approx 61.66 P_{\rm orb}^2 \qquad (16)$$

where $a$ is necessarily expressed in units of $R_\odot$ and $P_{\rm orb}$ in days. Inserting the result given by equation (16) into equation (11), we obtain:

$$M_1 + M_2 \approx \frac{61.66}{75} M_\odot \approx 0.82 M_\odot \qquad (17)$$

We can interpret this result as being the average typical value for the total mass of a WD-WD pair, which is in agreement with the values present in Table 1, since from these one gets an average total mass of 0.86 $M_\odot$.

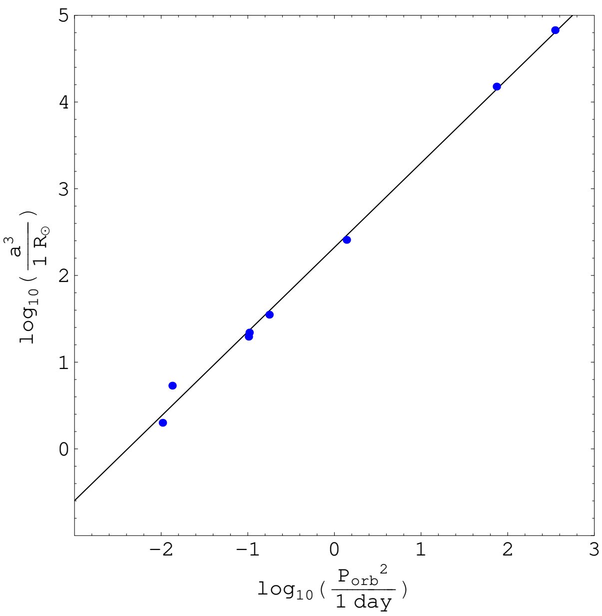


Figure 5: log-log relation between $a^3$ as a function of $P_{\rm orb}^2$ for all the 8 NS-NS pairs from Table 2.

In Figure 5, we show $a^3$ as a function of $P_{\rm orb}^2$ for all the 8 pairs from Table 2. After linear regression, we get:

$$\log a^3 \approx 0.9735 \log P_{\rm orb}^2 + 2.324 \qquad (18)$$

from which we get:

$$a^3 \approx 210.863 \times (P_{\rm orb}^2)^{0.9735} \tag{19}$$

Combining equations (19) and (11), we obtain:

$$M_1 + M_2 \approx \frac{2.81}{P_{\rm orb}^{0.053}} M_\odot \tag{20}$$

Since $P_{\rm orb}^{0.053} \approx 1$ for all cases of interest, we may write:

$$M_1 + M_2 \approx 2.81 M_\odot \tag{21}$$

which can be regarded as the average mass of a typical NS-NS system. Notice that this value agrees with the total masses shown in Table 2, from which we get the average value of 2.63 $M_\odot$.

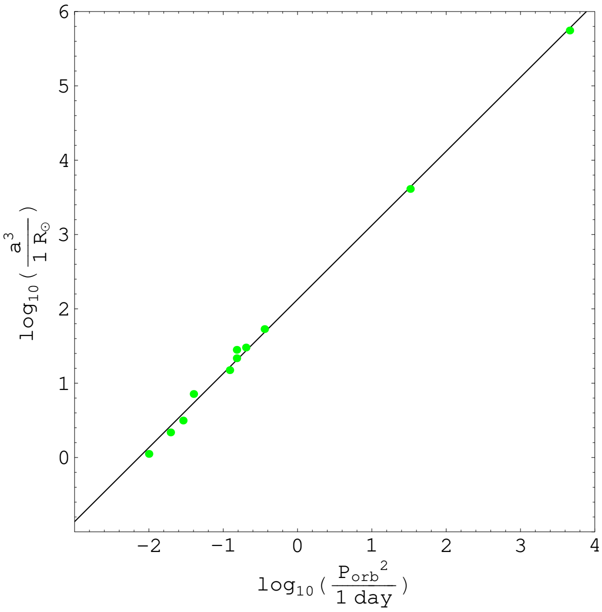


Figure 6: log-log relation between $a^3$ as a function of $P_{\rm orb}^2$ for all the 11 WD-NS cases shown in Table 3.

Similarly, in Figure 6, we show $a^3$ as a function of $P_{\rm orb}^2$ for all the 11 WD-NS cases shown in Table 3. After linear regression, we get:

$$\log a^3 = 0.996 \log P_{\rm orb}^2 + 2.127 \tag{22}$$

from which we get:

$$a^3 \approx 133.9 P_{\rm orb}^{1.992} \tag{23}$$

Notice that $P_{\rm orb}^{1.992} \approx P_{\rm orb}^2$. Inserting this result into equation (11), we have:

$$M_1 + M_2 \approx \frac{133.9}{75} M_\odot \approx 1.785 M_\odot \tag{24}$$

which can be interpreted as the average mass of a WD-NS pair. In fact, this value agrees with the average of the total masses of WD-NS pairs presented in Table 3, which is 1.83 $M_\odot$.

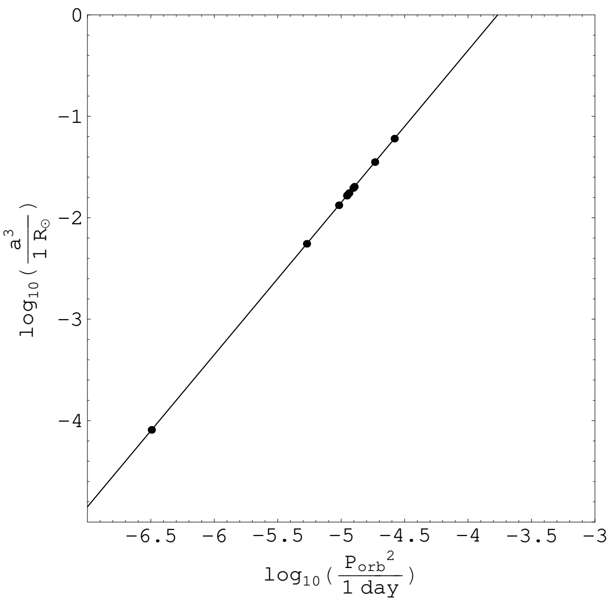


Figure 7: log-log relation between $a^3$ as a function of $P_{\rm orb}^2$ for all the 10 BH-BH pairs from Table 4.

In Figure 7, we show $a^3$ as a function of $P_{\rm orb}^2$ for all the 10 BH-BH pairs from Table 4. After linear regression we have:

$$\log(a^3) = 1.5 \log(P_{orb}^2) + 5.65 \tag{25}$$

from which we get:

$$a^3 = 4.47 \times 10^5 P_{orb}^3 \tag{26}$$

Inserting this last result into equation (11) we get:

$$M_1 + M_2 \approx 6 \times 10^3 P_{orb} \tag{27}$$

These results suggests that knowing the value of $P_{orb}$ in a close BH-BH system we can estimate the respective total mass. For example, for a close BH-BH system with an orbital period of 5 minutes ($P_{orb} \approx 0.0035$ days) we get $M_1 + M_2 \approx 21 M_\odot$.

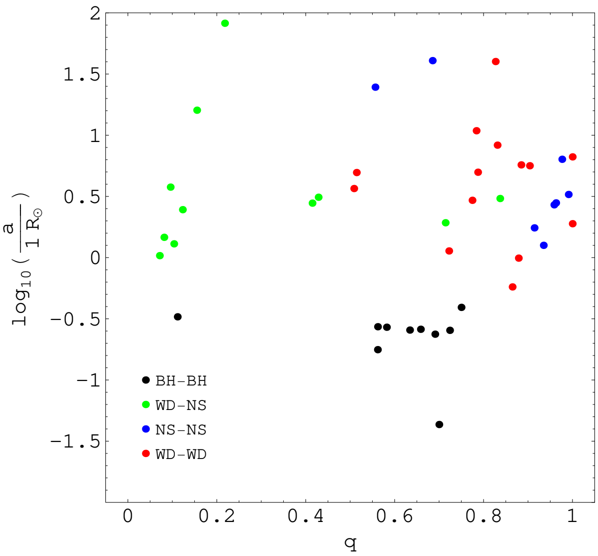


Figure 8: Relation between $\log_{10}(a)$ and $q$ for all the 43 double compact binaries from Tables 1, 2, 3 and 4.

With Figure 8, we tried to explore if there is some kind of functional relation between the semi-major axis $a$ and the mass ratio $q$ (cf. equation 2) for all the 43 compact binary systems from Tables 1, 2, 3 and 4.

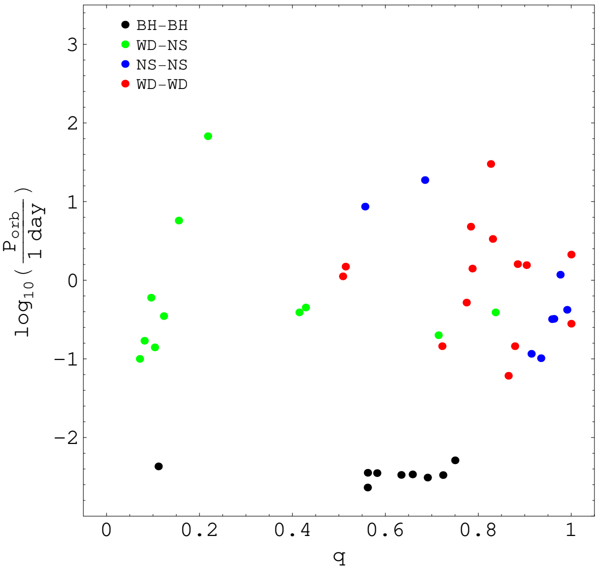


Figure 9: Relation between $\log_{10}$(a) and $q$ for all the 43 double compact binaries from Tables 1, 2, 3 and 4.

In Figure 9, we tried to do the same, but now, between $P_{orb}$ and $q$. A direct inspection of both Figures indicates that there is not an obvious relation between the two parameters. WD-WD and NS-NS binaries are mixed among each other and located on the right side of both Figures. This was expected since in this case we have binary systems with both components sharing a similar mass. The majority of the considered NS-WD binaries are located on the left side of both Figures ($q \approx 0.1$), since in this case we have, in general, components with quite different masses. It is interesting to notice that a few of the NS-WD pairs do not share this particularity, since that, they are located along an horizontal region ($a \approx 10^{0.5} R_{\odot}$ - see Figure 8). The 10 BH-BH are located in the lower part of Figures 8 and 9 since they have the lowest values of $a$ and $P_{orb}$ among all the considered binaries in our sample.

# 5 Conclusions

In general, a star will evolve to a compact object such as a WD, a NS or a BH. For an isolated star, this end product depends solely on the initial mass of that particular star. Considering that, approximately, half of all stars in our galaxy are in binary systems, it is reasonable to expect an important number of Double Compact Stars (DCSs) as a result of the late evolution of such binaries. In the case of a close binary system, the evolution of each star could follow a somehow different path than the one followed by a similar isolated star.

We searched the literature for examples of binary systems involving two compact stars. We obtained 14 WD-WD systems, 11 WD-NS systems, and 8 NS-NS systems. In each case, we compiled the respective masses, orbital period and semi-major orbital axis (see Tables 1, 2, 3). In the case of BH-BH binaries, there is no record of the observation of such binaries (i.e. binaries with BHs of stellar mass) other than those associated with the merger of BHs with the consequent emission of GWs. We then looked at the known cases of BH-BH mergers and tried to reconstitute the binary system at a stage when it was still relatively stable (we did this for the 10 systems with the lowest total mass so that we could compare them with the other types of binary systems under consideration - see Table 4). In total, we considered a sample of 43 DCSs. We did not include lists with binaries of the kind NS-BH and WD-BH since the few cases we found in the literature still reveal a lot of uncertainty regarding the nature of the two components of the binary.

By linear regression, we established the mean total mass for each kind of DCS system: $0.82M_\odot$ for WD-WD pairs, $2.81M_\odot$ for NS-NS pairs and $1.785M_\odot$ for NS-WD pairs. In the case of BH-BH the total mass, for the 10 considered cases, is proportional by a factor of $6 \times 10^3$ to $P_{orb}$. We found that there is a relation $P_{orb} \approx (a^{1.5})/10$ valid for WD-WD, WD-NS and NS-NS systems, which is, in fact, an expression of Kepler's Third Law.

In terms of future work, it is intended to populate the sample presented in this work with more compact binary star systems after performing an exhaustive literature revision. It is also intended to add more parameters (e.g. distances, luminosity, orbit eccentricity, star radius) and study the possible relations between those parameters.

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